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Dear Dr.Hratch Semerjian and Dr. Shyam Sunder: This is to respond the NIST WTC Team call for improvements of the draft report. This report has no merits because it is based on the wrong analysis of Bazant and Zhou attached herewith as a reminder. The correct explanation is given in my paper attached herewith, too. Regards Genady Cherepanov

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On the WTC Collapse

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Abstract: The generally-accepted explanation of the collapse of the World Trade Center towers on September 11, 2001 is based on the speculative "theory" of progressive buckling of bearing columns at the speed of free fall triggered by creep buckling of the columns of the floor subject to the conflagration from the spilled fuel, and by dynamic impact of the upper structure. In the present paper it is shown that this "theory" is wrong because it is built on false assumptions and incorrect calculations. The "theory" cannot explain the free fall, explosion sound, and pulverization of the buildings as well as other facts of this event. The simultaneous collapse of the neighboring 47-story tower directly contradicts to the "theory". It is shown that, consistent with all known facts of the matter, the scenario of all collapses was this: (i) heating of bearing columns in the "hot" spot caused high compressive thermal stresses in these columns, (ii) these stresses combined with internal stresses triggered a fracture wave, and (iii) the fracture wave disintegrated the entire building for less than 0.1 s producing the sound of explosion and providing the conditions necessary for free fall of steel fragments and dust clouds of tiny fragments of glass, marble and concrete. The theory of fracture waves supports this scenario.

Keywords: World Trade Center, tower, building, column, collapse, explosion, structure free fall, debris, structure pulverization, fracture wave, thermal stresses, internal stresses, creep, buckling, dynamic impact, progressive failure, triggering mechanism, accurate vs. approximate analysis.

Contents

1. Introduction

"Why did you think the towers collapse?", Larry asked his guest, a prominent member of the September 11 Commission, on a recent Larry King show. "This is still under investigation", the guest answered. Evidently, the public has not, as yet, accepted the "theory" by Bazant and Zhou (2002). Meanwhile, the engineering community has, without any hesitation, recognized the "theory" as correct and comprehensive. This author has felt this official recognition on his own skin after the editors and anonymous referees of numerous technical journals refused to publish his understanding of the collapses as contradicting to this "theory".

The "theory" has suggested the following scenario of the collapse: creep buckling of bearing columns of the critical floor, free fall and dynamic impact of the upper structure, and progressive, floor-by-floor, buckling failure of bearing columns of the underlying structure. This "theory" has been unable to explain these well-known facts of the matter:

(i) Free fall regime of all collapses;

(ii) Sound of explosion produced by each collapse;
(Sound is generated by cracking. If the cracking had continued for ten seconds, as the "theory" asserts, a boom would have been heard, not an explosion.)

(iii) Pulverization of the buildings collapsed .
(By the "theory" the debris after the collapse would have consisted of steel segments of columns about two meters long, and nothing more.)

According to the "theory" the neighboring 47-story building should NOT have collapsed. But, it did.

According to the "theory" the Empire State Building should have collapsed in 1945 under similar conditions of aircraft crash and conflagration. But, it did NOT.

Meanwhile, for every person familiar with industrial implosions, when a building is intentionally demolished by uniformly distributed explosives to produce small debris for their easier transport, the WTC collapse has strikingly resembled that of a designed implosion caused by previously distributed explosives. Indeed, each tower collapse took about ten seconds, that is all parts of each building were falling free, without any resistance. It is exactly what happens after a building is disintegrated by explosives. It is no wonder that the conspiracy theory, consistent with the well-known facts of the matter contrary to the official "theory", has become widely spread in the world.

In what follows, it is shown that the official "theory" is built on false assumptions and miscalculations, and hence wrong; and a scientific explanation consistent with all known facts is suggested.

2. Triggering mechanism: thermal stresses vs. creep

"A loss of protective thermal insulation of steel columns during the initial blast accelerated the heating of the columns to very high sustained temperature well above 800°C which lowered the yield strength and caused creep buckling of more than half of the columns in the critical floor, so that the upper part of the structure above this floor fell down and, by enormous vertical dynamic load, destroyed the underlying segment of the tower; and so the series of impacts and failures proceeded all the way down", the official "theory" says, when paying no attention to thermal stresses and residual technological stresses arisen from rolling, welding, assembling, etc. Amazingly, the "theory" ignores even the main event – the combustion of spilled fuel in the critical floor, the event that caused all the collapses.

All assumptions and claims of this "theory" are false. First, the loss of the protective thermal insulation of more than half of the 260 columns of the critical floor by the initial blast is nothing but a miracle necessary for the "theory" because for creep time is essence. Remember that the time between each crash and collapse took about one hour which was, by itself, a very little time for a creep action in a steel column at the level of stresses, at least, three times less than the yield strength and/or the buckling stress at normal temperature, due to the safety factor, even if the entire lateral surface of the column was exposed to the temperature 800°C all this time.

The rate of heat propagation is controlled by the thermal diffusivity, which is equal to $12\times10^{-6}\,m^2/s$ for steel and about a fifty times less for the protective thermal insulation. How fast is this process in terms of time? Let us provide an accurate example. Suppose the initial temperature of a steel half-space is zero. It takes one hour to increase the temperature to 650°C at the distance 8 cm from the surface kept at 800°C all this time. For the thermal insulation, the corresponding distance is about 1 cm, all other conditions being the same. In other words, one hour is about the time necessary for the heat to penetrate through the protective thermal insulation of a bearing column; it takes one more hour to warm up the column itself. There is no time for creep action.

Secondly, the assumption that 800°C was the temperature of four-meter-long bearing columns of the critical floor during the fire, is quite frivolous. Again, let us examine an example of accurate calculation. Suppose n-octane fuel is burned in the constant pressure, adiabatic combustor of an aircraft engine with 40% excess air, and the fuel is injected into the combustor at 25°C while the air from the compressor enters this combustor at 600 KPa, 300°C. One can find that the combustion products leave the combustor for the turbine at the temperature 769°C, so that the mean temperature of turbine blades is well below 700°C. These are the real conditions of the fuel combustion in the engines of the Boeings that crashed into the towers.

Let us compare the combustion of the fuel spilled in the critical floor of the WTC tower with the combustion of this fuel in the Boeing engine. The combustor will be the whole floor, open-to-air, space with a liquid fuel layer on the bottom, with the air entering this combustor from the atmosphere at 100 KPa, 25°C. Compare the temperature of the Boeing turbine blades with that of thermally-protected columns of the floor. The combustion in the engine runs under the perfect conditions of homogeneous turbulence in a homogeneous mixture designed to achieve the temperature of combustion products as high as possible. The combustion in the open, non-adiabatic floor is, evidently, incomplete, far from the stoichiometric balance, with cold air and a low air-fuel ratio, with the reaction taking place in convective flames providing a very non-uniform distribution of temperature in space and time. For example, the temperature of the tip of the convective flame of a candle can achieve 500°C but you can put it out with a finger because the mean temperature of the flame is below 100°C. And so, the mean temperature in the burning surroundings of the bearing columns was probably below 500°C while locally, at some spots close to the ceiling of the floor, it could achieve 1000°C and higher because of high adiabatic flame temperatures of the fuel. For creep buckling to be true, the entire column should be at a high temperature for a long time.

Thirdly, the decrease of the yield strength of steel was too little to play any role in the collapses. Structural hot-rolled steel used in columns has the yield strength about 600 MPa and the ultimate strength about 900 MPa, at 20°C. At 800°C the numbers are 10 to

20% lower while the nominal stress in columns was, at least, three times less than the yield strength.

From this analysis of conflagration, it follows that the claim of creep buckling of the "theory" is groundless. A measurable creep of structural austenitic steels starts from about 540°C. Meanwhile, this and higher temperatures could be achieved only locally, in the top parts of some bearing columns where the flame temperature was maximal. And because of the thermal protection, these temperatures could be sustained during some time much less than one hour.

For the "theory", it is essential that each bearing column of the floor should be, from the bottom to the top, heated to one and same high temperature sustained for a long time, because in the case of uniform heating of all columns there are no thermal stresses in the columns, so that the thermal stresses can be ignored. If only some of the columns are heated, the thermal stresses arise that can achieve an order of αET where α is the thermal expansion coefficient, E is Young's modulus, and T is the temperature. For steel $\alpha = 12\times10^{-6}$ /°C and $E = 200$ GPa so that at 800°C the thermal stress can achieve 2 GPa which is about four times greater than the yield strength of steel at 800°C.

The calculation of the time-space distribution of temperature and thermal stresses in a building under the real conditions of a fire is a delicate procedure responsible for providing a correct prediction or explanation of a final outcome. Whether a building would collapse or be preserved depends on the thermal stress distribution. Any material volume or structure will be torn into pieces by thermal stresses if some part of the structure is heated too fast to a high temperature.

Just for the purpose of rough estimate, let us do some calculations using the notion of a "hot spot" inside the building. The bearing columns in the hot spot are heated to one and same temperature T while the bearing columns outside the hot spot retain the initial temperature $T = 0$. And so, the thermal stresses in the hot columns are compressive while in the cold columns they are tensile. In the case of the conflagration in the WTC towers and adjacent 47-story building, the core columns were probably in the hot spot while, at least, some bearing columns of framed tube cooled by atmospheric air were outside the hot spot. Compressive thermal stresses, being diffused only by bending floor trusses and cold columns of framed tube, penetrated far into cold columns of the underlying structure. Combined with gravitational and residual technological stresses, the compressive thermal stresses inside the building created a ticking bomb like that of a Batavian tear, so that a fracture wave was born that disintegrated the entire tower for less than 0.1 s.

Let us remind that a Batavian tear, just taken from a glass bath and treated by fluoric acid to dissolve the cracked surface layer, has a core under high compressive stresses and a flawless surface layer under high tensile stress about 5 GPa. Breaking the tiny tail on the Batavian tear releases the elastic energy of compressive stresses in a fracture wave that propagates at the speed of sound and pulverizes glass into micron-size fragments. (See Appendix) Also, as a reminder the compressive residual stress from rolling in steel columns can achieve a half or more of the yield strength.

Suppose S_A is the cross-section area of all bearing columns of the critical floor. Let us assume that βS_A is the cross-section area of the hot bearing columns heated to the temperature T and $(1-\beta)S_A$ is the cross-section area of cold bearing columns at the temperature $T = 0$. As a result, the hot columns will be subject to the compressive thermal stress

$$\sigma = -\delta(1-\beta)\alpha ET \quad \text{where } 0<\beta<1,\ \tfrac{1}{2}<\delta<1, \tag{1}$$

while the cold columns will be subject to the tensile thermal stress

$$\sigma = \delta\beta\alpha ET \quad \text{where } 0<\beta<1,\ \tfrac{1}{2}<\delta<1. \tag{2}$$

The coefficient δ takes into account the elastic reaction of the ends of columns. For rigid floor trusses $\delta = 1$, and for very soft floor trusses, when the elastic reaction of supports is created by the columns themselves, $\delta = 0.5$. And so, the hot columns will be under action of the sum of compressive gravitational and thermal stresses while the cold columns will be unloaded by the thermal stresses. In this illustrative estimate, we ignore residual stresses.

A collapse can start either from tensile failure of cold columns or from the buckling of hot columns in the critical floor. Let us estimate the critical size of the hot spot for both cases.

Suppose that the buckling of hot columns occurs at $\beta = \beta_b$ and that $-f\sigma_Y$ is the nominal stress in all columns of the floor from the weight of the upper structure, where f is the safety factor and σ_Y is the yield strength of steel. Let $-f_o\sigma_Y$ be the stress in hot columns when the buckling occurs, where $f_o \geq f$ evidently. From here and equation (1) it follows that

$$f\sigma_Y + \delta(1-\beta_b)\alpha ET = f_o\sigma_Y, \tag{3}$$

and

$$\beta_b = 1 - \frac{(f_o - f)\sigma_Y}{\delta\alpha ET}. \tag{4}$$

Now, suppose that the failure of cold columns from tensile stresses occurs at $\beta = \beta_T$. From here and equation (2), it follows that

$$\delta\beta_T\alpha ET - f\sigma_Y = \sigma_b, \tag{5}$$

and

$$\beta_T = \frac{\sigma_b + f\sigma_Y}{\delta\alpha ET}, \tag{6}$$

where σ_b is the ultimate tensile strength of structural steel. Make the ratio β_b / β_T from equations (4) and (6)

$$\frac{\beta_b}{\beta_T} = \frac{\delta\alpha ET - f_o\sigma_Y + f\sigma_Y}{\sigma_b + f\sigma_Y}. \tag{7}$$

From equation (7) it follows that

$$\frac{\beta_b}{\beta_T} > 1 \qquad \text{because} \qquad \delta\alpha ET > \sigma_b + f_o\sigma_Y. \tag{8}$$

For example, for typical values when $\alpha ET = 2\,\text{GPa}$, $\sigma_Y = 0.5\,\text{GPa}$, $\sigma_b = 0.7\,\text{GPa}$, $f_o = 0.5$, $f = 0.25$, and $\delta = 0.75$, we get $\beta_b / \beta_T = 5/3$.

It means that the collapse started from tensile failure of cold columns because the critical size of the hot spot in this scenario was less than that in the scenario of the buckling of hot columns. The hot spot was evidently expanding during the fire.

And so, the failing cold columns of the critical floor played the role of a tiny tail of a Batavian tear that explodes into small fragments when the tail is broken. The failure of the cold columns of the critical floor started the process of release of elastic energy of compressive stresses that occurred in a fracture wave because it is only the fracture wave that can pulverize material.

3. Dynamics: accurate vs. approximate analysis

According to the "theory" the upper part of the tower above the critical floor freely fell down in the beginning of the collapse and created an "enormous" dynamic stress in the bearing columns of the underlying structure, so that the maximum dynamic stress was 64.5 times greater that the nominal static stress in these columns from the weight of the upper structure. "This estimate is calculated from the elastic wave equation", the "theory" says.

Let us verify this calculation. Suppose mass m falls down under gravitational force and hits the end of a vertical elastic column or bar at the speed V_o and sticks to the end. It is easy to find the material velocity v_x and stress σ_x in the column/bar arising from this impact:

$$v_x = \frac{mg}{SE}c + \left(V_o - \frac{mg}{SE}c\right)\exp\left[\frac{SE}{mc^2}(x - ct)\right], \tag{9}$$

$$\sigma_x = -\frac{mg}{S} + \left(-\frac{V_o}{c}E + \frac{mg}{S}\right)\exp\left[\frac{SE}{mc^2}(x - ct)\right]. \tag{10}$$

Here: $0 < x < ct$; t is the time from the moment of impact $t = 0$; x is the coordinate along the bar located at $x > 0$; E is Young's modulus and c is the speed of elastic waves in the column equal to $\sqrt{E/\rho}$ where ρ is the density; and S is the column cross-section area. For $x > ct > 0$ both σ_x and v_x equal zero.

In particular, at the end of the column at $x = 0$ $t > 0$, the stress and velocity are:

$$\sigma_x = -\frac{mg}{S} + \left(-\frac{V_o}{c}E + \frac{mg}{S}\right)\exp\left[-\frac{SE}{mc}t\right], \tag{11}$$

$$v_x = \frac{mg}{SE}c + \left(V_o - \frac{mg}{SE}c\right)\exp\left[-\frac{SE}{mc}t\right]. \qquad (12)$$

The maximum stress is equal to:

$$\sigma_x = -\frac{V_o}{c}E \quad \text{when} \quad x = 0 \quad t = 0. \qquad (13)$$

If the assumption of the "theory" about free fell of the upper structure is accepted, then $V_o = \sqrt{2gh} = 8.5$ m/s because the height of the floor $h = 3.7$ m and $g = 9.8$ m/s^2. For steel columns, $c = 5.1$ Km/s and $E = 200$ GPa, so that according to equation (13) the maximum stress in the columns of the underlying structure is equal to 340 MPa. Based on the indicated estimate of the "theory" the nominal static stress in these columns, that is mg/S, should be equal to 340/64.5=5 MPa which is a hundred times less than the yield strength of steel. It is unbelievable! Even a teen girl can produce such a pressure on the floor by her high heels. The approximate estimate of the "theory" is very inaccurate.

However, even the maximum stress 340 MPa from the impact, greatly exaggerated due to the free fall assumption, is about six times less than the maximum thermal stress 2 GPa. And so, the role of dynamic overload from the impact of the upper structure turns out to be secondary as compared to the thermal stresses. The dynamic stress could contribute to the compressive thermal stresses of the underlying columns to mutually create a fracture wave, if these columns had not been disintegrated still earlier by a fracture wave. The time of free fall of the upper structure for the height $h = 3.7$ m equals $\sqrt{2h/g} = 0.75$ s which is much greater than the time 0.05 s necessary to disintegrate the whole building by a fracture wave if it was created immediately after the tensile failure of cold bearing columns.

By the way, the authors of the "theory" missed the fact that the maximum dynamic stress would travel all the way down at the speed 5 Km/s and that the fracture wave of disintegration should immediately follow the shock wave of compression because no material could bear the "enormous" compression stress that was, according to the theory, 64.5 times greater than the static stress. And so, the "theory" supports the fracture wave mechanism of the collapses, not the progressive failure mechanism. But, what happened is more complicated than what implied by the "theory".

Beyond the present calculation of dynamic overload, there is direct evidence that it is the thermal, not dynamic, stress that triggered the collapse of the neighboring 47-story tower. A portion of spilled fuel got on the top of the latter building and set a fire there. There were no upper structure above to fall down and start the collapse as the "theory" claims. It is only the thermal stresses that could trigger a fracture wave of disintegration in this case.

4. Free fall: fracture wave vs. progressive failure

To explain the free fall regime of the collapses, the "theory" assumes that at any moment of collapse there are exist an upper part of the tower that moves down and an underlying structure that rests intact, and that the underlying structure produces no reaction and resistance to the falling upper part because "the inelastic energy dissipation in plastic hinges of collapsing columns is much less than the kinetic energy of the falling mass".

This thesis is an evident blunder. The loss of kinetic energy of the falling mass is caused, mostly, by the elastic deformation of the underlying structure, and the resistance of a solid structure is due, mostly, to the elastic reaction that can stop the falling mass even if the inelastic energy dissipation is zero. For example, the "enormous" dynamic overload from the impact of the upper structure on the critical floor, which is according to the "theory" 64.5 times greater than the static load, should be also applied to the moving mass creating the force of resistance, by the Newton law, which is disregarded by the "theory".

Even within the framework of progressive failure model, the inelastic energy dissipation was miscalculated. It is true that the energy dissipated in plastic hinges of buckling columns of the underlying structure is about 8.4 times less than the decrease of the gravitational energy of the upper structure falling down in the critical floor. However, it is valid with account of only one plastic hinge per column of one floor, which contradicts to the following facts. First, the dynamic instability of columns/bars occurs by higher order modes of buckling (the greater is the dynamic load, the higher is the mode of buckling). Secondly, the debris should be two-meter-long segments of columns, which is very far from the reality. The same calculation would predict the ratio 2.8, and not 8.4, if three plastic hinges per column of one floor would be taken into account. In this case the debris would be one-meter-long segments of columns, which is closer to the reality. Any accurate calculation would show that the inelastic energy dissipation during the collapse is significant and comparable with the decrease of gravitational energy and the value of the corresponding kinetic energy.

Let us analyze the model of "progressive failure" avoiding the mistakes of the "theory". Suppose that all columns of the critical floor disappeared and the upper structure freely fell down on the underlying structure, as suggested in the "theory". From the accurate solution of Section 3 it follows that the maximum total stress in the columns of the underlying structure from the impact is equal to 340 MPa which is almost twice less than the yield strength of steel. This value must be close to the buckling stress of well-designed columns, with account of the safety factor. Taking into consideration that 340 MPa is greatly exaggerated by the free fall assumption and that this maximum stress is kept for a quite short time much less than about 0.01 s, it is doubtful that this improvised impact could produce any fracture or failure in the columns of the underlying structure. The buckling failure could be possible only in the case of very flexible columns of a very bad design because the buckling stress of even flexible columns is several times greater for the dynamic load than that for the static load due to higher modes of buckling.

Hence, the progressive failure is nothing but a result of the miscalculations of the "theory".

The only possible scientific explanation of the free fall regime of the collapses is that the buildings were disintegrated by fracture waves at the beginning of each collapse, which took about 0.05 s because fracture waves propagate at the speed about 6 Km/s in steel, glass, concrete, and marble. The disintegration by cracking is unnoticeable for such a short time because the volume of cracks is very small as compared to the volume of intact material, with no visible deformations during that time. The cracking of the tower for 0.05 s produced the sound emission heard as an explosion. A boom would be heard if the cracking took 10 s as suggested by the "theory" of progressive failure. For a fracture wave to propagate, a material should be loaded by compressive stresses of high energy because this energy is released in the fracture wave. (See Appendix).

The material velocity of fragments behind the fracture wave has an order of 10 to 100 m/s depending on material and stress; for glass it is about four times greater than for steel. The size of fragments behind the fracture wave depends on stress and material; for steel it is about 5 to 50 cm, and for glass, concrete and marble it is about 0.1 to 10 μm. Combination of free gravitational fall of heavy steel fragments and explosive sweep-away of particles of glass, concrete and marble in the form of dust clouds created the picture of the collapses observed on TV screens.

A classical example of the fracture wave action is a Batavian tear of glass. If one breaks a tiny tail on the Batavian tear, it explodes into a cloud of dust with a loud sound. It takes 10^{-5} s to pulverize a five-centimeter tear by a fracture wave and 10^{-2} s to create a one-meter cloud of micron-size particles of glass.

And so, the fracture wave mechanism of the WTC collapse and the collapse of the neighboring 47-story building is supported by the following facts:

(i) All buildings collapsed in free fall regime;

(ii) Each collapse was accompanied by a sound of explosion;

(iii) The size of steel fragments and dust particles of glass, concrete and marble corresponds to that calculated in the theory of fracture waves;

(iv) Dust particles created clouds expanded for several hundred meters.

5. Fracture wave vs. shock wave

Let us summarize the basic properties of shock waves and fracture waves following Cherepanov (1979). Both waves represent some fronts of discontinuity of material density, velocity, and stresses.

Shock waves are produced by impacts and explosions in gases, liquids, and solids. The density of material behind a shock wave is always greater than in front of the wave. The maximum compressive stress behind a shock wave is always greater than in front of the wave. The normal velocity of a shock wave is always greater than the speed of sound (in solids and liquids, slightly greater). The thickness of a shock wave is defined by viscous properties of a material.

It is a widely-spread but wrong belief that a shock wave can disintegrate a material into small fragments*. To disintegrate means to crack, but a shock wave cannot crack a solid because any cracking is accompanied by a dilatation of the solid. A fracture wave should always follow a shock wave in order to disintegrate a material.

Fracture waves can be produced only by compressive stresses in solids. Fracture wave separates an intact material in front of the wave from a destructed material behind the wave. The thickness of a fracture wave has an order of the size of material fragments behind the wave. The mean density of a material behind a fracture wave is always less than in front of the wave. The maximum compressive stress behind a fracture wave is always less than in front of the wave. The normal velocity of steady fracture waves is

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* Dr. Bazant and many anonymous referees have stuck to this opinion.

equal to the speed of sound (longitudinal elastic wave). For unsteady fracture waves, the normal velocity is less than the speed of sound and determined from the solution of a particular problem, that is, depends on boundary and initial conditions.

6. Conclusions

It was shown that, in the tragic collapses on September 11, 2001:

(i) Creep played no part, and these were the thermal stresses that triggered the collapses;

(ii) Tensile failure of some cold bearing columns from the thermal stresses started the collapses, and not the creep buckling of hot columns;

(iii) Dynamic stress from the impact of the upper structure on the initial stage of each collapse was insufficient even to produce a failure of the underlying structure, not to say about a progressive failure of entire buildings;

(iv) A fracture wave, originated after tensile failure of some cold bearing columns in the critical floor, disintegrated each building for about 0.05 s and produced the sound of explosion, and steel fragments freely fell down while glass, concrete and marble fragments created dust clouds.

The fracture wave mechanism is the most plausible hypothesis because it is supported by the facts of the matter and by accurate calculations. However, the exact conditions triggering fracture waves need to be studied which is a challenging problem for the future.

Acknowledgement

This author thanks the editors and anonymous referees of the technical journals that rejected the author's explanation because it contradicted to the official "theory". Their comments have stimulated this author to undertake the present analysis of the "theory" that many trust to so deeply.

References

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2. Genady P. Cherepanov (1979) Mechanics of Brittle Fracture, McGraw-Hill, New York, p. 1-980.

Appendix. The theory of fracture waves

The fracture wave is a front of discontinuity of mass density, material velocity and stresses that separates an intact material in front of the fracture wave from a destructed one behind. The mass density behind a fracture wave is always less than that in front of the wave because any cracking of a solid dilates it. The thickness of a fracture wave has an order of the size of fragments of the destructed material behind the wave.

The conservation laws on the fracture wave can be written as follows:

mass conservation

$$\rho_0(V - v_0) = \rho_F(V - v_F), \tag{A.1}$$

momentum conservation

$$-\sigma_0 + \rho_0(V - v_0)^2 = -\sigma_F + \rho_F(V - v_F)^2, \tag{A.2}$$

energy conservation

$$\frac{1}{2}(V - v_0)^2 + \frac{U_0}{\rho_0} - \frac{\sigma_0}{\rho_0} = \frac{1}{2}(V - v_F)^2 + \frac{U_F}{\rho_F} - \frac{\sigma_F}{\rho_F} + \frac{D}{\rho_F}. \tag{A.3}$$

Here: lower index 0 refers to the intact material in front of the fracture wave, lower index F refers to the destructed material behind the fracture wave, V is the normal velocity of the fracture wave, v is the material velocity normal to the fracture front, ρ is the material density, U is the volume density of elastic energy of the material, σ is the stress component normal to the fracture front, D is the volume density of surface energy of the destructed material.

Equations (A.1) and (A.3) can be re-written as follows:

$$\frac{1}{\rho_0} - \frac{1}{\rho_F} = \frac{1}{\rho_0}\frac{v_F - v_0}{V - v_0}, \tag{A.4}$$

$$\sigma_0 - \sigma_F = \rho_0(V - v_0)(v_F - v_0), \tag{A.5}$$

$$\frac{D}{\rho_F} = \frac{U_0}{\rho_0} - \frac{U_F}{\rho_F} + \frac{1}{2}(\sigma_0 + \sigma_F)\left(\frac{1}{\rho_F} - \frac{1}{\rho_0}\right). \tag{A.6}$$

Let us assume that the intact material is at rest, i.e., $v_0 = 0$. Then, the values of ρ_F, v_F and D can be found from equations (A.4) to (A.6) as follows:

$$\rho_F = \frac{\rho_0}{1 - \dfrac{\sigma_0 - \sigma_F}{\rho_0 V^2}}, \tag{A.7}$$

$$v_F = \frac{\sigma_0 - \sigma_F}{\rho_0 V}, \tag{A.8}$$

$$D = \frac{\rho_F}{\rho_0}\left(U_0 - \frac{\sigma_0^2 - \sigma_F^2}{2\rho_0 V^2}\right) - U_F. \tag{A.9}$$

From equations (A.7) and (A.8), it follows that $v_F < 0$ and $\sigma_0 < 0$ because $\rho_0 > \rho_F$ due to the physical meaning of the fracture wave. It means that the fracture wave can propagate only in a compressed material and the velocity of destructed material is always opposite to the normal velocity of the fracture wave.

Let us confine ourselves by steady fracture waves. Assume for a moment that $V < c$ where c is the speed of longitudinal elastic waves in the material. An elastic forerunning field ahead of such a fracture wave would also be steady-state. However, from the theory of elasticity it follows that steady elastic field can propagate only at the speed of c. (The shear wave is, evidently, impossible). It means the assumption is not valid, so that $V \geq c$ for steady fracture waves. From equation (A.7) it follows that ρ_F is very close to ρ_0, i.e. $\rho_F \approx \rho_0$ because $\sigma_0 << E$ and $\rho_0 V^2 \geq \rho_0 c^2 \approx E$. And so, equation (A.9) becomes

$$D = U_0 - \frac{\sigma_0^2}{2\rho_0 V^2} - \left(U_F - \frac{\sigma_F^2}{2\rho_0 V^2}\right). \tag{A.10}$$

Let us neglect the mutual contacts of fragments of the destructed material because of lost coherence, so that $\sigma_0 >> \sigma_F$ and $U_0 >> U_F$, and equations (A.8) and (A.10) take the form

$$v_F = \frac{\sigma_0}{\rho_0 V}, \qquad D = U_0 - \frac{\sigma_0^2}{2\rho_0 V^2}. \tag{A.11}$$

Let us analyze D as a function of V. Based on the principle of minimum of surface energy the value of D should be minimum possible because D is the surface energy of the destructed material in unit volume. From this principle, it follows that $V = c$, because D is minimal at $V = c$. In 1967, the same conclusion was derived by this author and Leo A. Galin based on the analogy between the fracture wave and detonation wave in TNT (the Chapman-Jouguet hypothesis).

And so, the basic equations of steady fracture waves can be summarized as follows:

$$V = c, \qquad D = U_0 - \frac{\sigma_0^2}{2\rho_0 c^2}, \qquad v_F = \frac{\sigma_0}{\rho_0 c}, \qquad \rho_F \approx \rho_0. \tag{A.12}$$

These equations are valid for any anisotropic, quasi-brittle materials whose dimensions are much greater than the thickness of the fracture wave, that is the size of fragments of the destructed material. Using the effective surface energy Γ of the cracking

of the material known from fracture mechanics tests, one can estimate the size of fragments of the destructed material in terms of Γ and D. E.g., one can find that:
if fragments are identical cubes with rib d,

$$d = 12\frac{\Gamma}{D}, \qquad \text{and} \tag{A.13}$$

if fragments are long identical needles of hexagonal cross-section with rib r,

$$2r = \frac{8}{\sqrt{3}}\frac{\Gamma}{D}. \tag{A.14}$$

The needle shape of fragments was observed in some experiments with glass specimens.

Suppose an isotropic material is in the state of hydrostatic compression by stress σ_0 in front of the fracture wave. In this case, we have

$$U_0 = \frac{3}{2}\frac{(1-2\nu)}{E}\sigma_0^2, \qquad \rho_0 c^2 = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}. \tag{A.15}$$

Here E and ν are Young's modulus and Poisson's ratio. Using equations (A.12) to (A.15) we get the following results for silicate glass at $\Gamma = 2$ N/m, $\rho_0 = 2.4$ g/cm^3, $E = 7\times10^4$ N/mm^2, and $\nu = 0.17$: $V = c = 5950$ m/s and
at $\sigma_0 = -500$ N/mm^2: $v_F = -35$ m/s, $D = 1.9$ N/mm^2, $d = 12.8\,\mu$m, $2r = 5\,\mu$m;
at $\sigma_0 = -1$ KN/mm^2: $v_F = -70$ m/s, $D = 7.5$ N/mm^2, $d = 3.2\,\mu$m, $2r = 1.2\,\mu$m;
at $\sigma_0 = -5$ KN/mm^2: $v_F = -350$ m/s, $D = 187.5$ N/mm^2, $d = 0.1\,\mu$m, $2r = 0.05\,\mu$m.

The glass needles in the range of $2r$ from about 1 μm to about 10 μm were observed experimentally, Cherepanov (1979). For rocks and building materials like concrete, marble, and wood the figures for v_F, D, d, and r are comparable to those in glass because their specific surface energy Γ is comparable with that of glass.

The dust produced by the collapses of three buildings on September 11, 2001 was created by micron-size fragments of glass, concrete and marble, in correspondence with these calculations because the thickness of fracture waves in these materials was much less than any structural dimension.

Suppose, now, that a fracture wave propagates in a steel column between the bottom and ceiling of a floor. Suppose that the column is a solid, vertical, round cylinder and that the steel fragments behind the fracture wave represent some segments of the column cracked along sliding planes inclined at 45° to the axis of the cylinder. In this case the height of the segment h_s is equal to

$$h_s = 2\sqrt{2}\frac{\Gamma}{D}, \tag{A.16}$$

and

$$U_0 = \frac{\sigma_0^2}{2E}, \qquad \rho_0 c^2 = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} . \tag{A.17}$$

Here σ_0 is the mean compressive stress in the intact segment in front of the fracture wave from gravitational, thermal and technological stresses (e.g. from rolling, welding, and assembling). The fracture wave releasing the potential energy of compressive stresses outstrips the group speed $\sqrt{E/\rho_0}$ so that at the distance of 4 m, the height of the column, it goes ahead by about 0.3 m.

Using equations (A.12), (A.16) and (A.17) one can find for steel: at $\Gamma = 20$ KN/m, $\rho_0 = 7.9\,\mathrm{g/cm^3}$, $E = 200\,\mathrm{GPa}$, and $\nu = 0.33$: $V = c = 5850\,\mathrm{m/s}$ and

at $\sigma_0 = -1$ KN/mm^2: $v_F = -21\,\mathrm{m/s}$, $D = 0.83\,\mathrm{N/mm^2}$, $h_s = 6.8\,\mathrm{cm}$;

at $\sigma_0 = -500\,\mathrm{N/mm^2}$: $v_F = -10\,\mathrm{m/s}$, $D = 0.2\,\mathrm{N/mm^2}$, $h_s = 27.2\,\mathrm{cm}$.

It should be noted that the effective surface energy Γ of steel includes the plastic energy dissipated in a thin layer on the crack surface. And so, the rough estimate of the size of steel debris based on the accurate energy balance in the fracture wave provides a realistic picture relevant to the collapses of all three buildings on September 11, 2001 because h_s is much less than the height of a column in a floor.

Another approach to the estimate of steel debris created during the collapses of the buildings is to model the building as a solid material volume of the same mass and shape, structurally orthotropic with vertical axis of symmetry and polar planes of symmetry, whose stiffness in these directions is equal to the stiffness of the building. The effective surface energy of this model material is equal to $(1-\varepsilon)^{\Gamma}$ where ε is the ratio of the empty space volume to the volume of the building, and Γ is the effective surface energy of steel. The propagation of fracture waves in porous materials requires a similar approach.

RAPID COMMUNICATION

Why Did the World Trade Center Collapse?—Simple Analysis[1]

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Abstract: This paper presents a simplified approximate analysis of the overall collapse of the towers of World Trade Center in New York on September 11, 2001. The analysis shows that if prolonged heating caused the majority of columns of a single floor to lose their load carrying capacity, the whole tower was doomed.

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CE Database keywords: New York; New York City; Disasters; Buildings; High-rise; Collapse; Analysis; Terrorism.

Introduction and Failure Scenario

The 110-story towers of the World Trade Center were designed to withstand as a whole the forces caused by a horizontal impact of a large commercial aircraft (Appendix I). So why did a total collapse occur? The cause was the dynamic consequence of the prolonged heating of the steel columns to very high temperature. The heating lowered the yield strength and caused viscoplastic (creep) buckling of the columns of the framed tube along the perimeter of the tower and of the columns in the building core. The likely scenario of failure is approximately as follows.

In stage 1 (Fig. 1), the conflagration, caused by the aircraft fuel spilled into the structure, causes the steel of the columns to be exposed to sustained temperatures apparently exceeding 800°C. The heating is probably accelerated by a loss of the protective thermal insulation of steel during the initial blast. At such temperatures, structural steel suffers a decrease of yield strength and exhibits significant viscoplastic deformation (i.e., creep—an increase of deformation under sustained load). This leads to creep buckling of columns (Bažant and Cedolin 1991, Sec. 9), which consequently lose their load carrying capacity (stage 2). Once more than half of the columns in the critical floor that is heated most suffer buckling (stage 3), the weight of the upper part of the structure above this floor can no longer be supported, and so the upper part starts falling down onto the lower part below the critical floor, gathering speed until it impacts the lower part. At that moment, the upper part has acquired an enormous kinetic energy and a significant downward velocity. The vertical impact of the mass of the upper part onto the lower part (stage 4) applies enormous vertical dynamic load on the underlying structure, far exceeding its load capacity, even though it is not heated. This causes failure of an underlying multifloor segment of the tower (stage 4), in which the failure of the connections of the floor-carrying trusses to the columns is either accompanied or quickly followed by buckling of the core columns and overall buckling of the framed tube, with the buckles probably spanning the height of many floors (stage 5, at right), and the upper part possibly getting wedged inside an emptied lower part of the framed tube (stage 5, at left). The buckling is initially plastic but quickly leads to fracture in the plastic hinges. The part of building lying beneath is then impacted again by an even larger mass falling with a greater velocity, and the series of impacts and failures then proceeds all the way down (stage 5).

Elastic Dynamic Analysis

The details of the failure process after the decisive initial trigger that sets the upper part in motion are of course very complicated and their clarification would require large computer simulations. For example, the upper part of one tower is tilting as it begins to fall (Appendix II); the distribution of impact forces among the underlying columns of the framed tube and the core, and between the columns and the floor-supporting trusses, is highly nonuniform; etc. However, a computer is not necessary to conclude that the collapse of the majority of columns of one floor must have caused the whole tower to collapse. This may be demonstrated by the following elementary calculations, in which simplifying assumptions most optimistic in regard to survival are made.

For a short time after the vertical impact of the upper part, but after the elastic wave generated by the vertical impact has propagated to the ground, the lower part of the structure can be approximately considered to act as an elastic spring [Fig. 2(a)]. What is its stiffness C? It can vary greatly with the distribution of the impact forces among the framed tube columns, between these columns and those in the core, and between the columns and the trusses supporting concrete floor slabs.

[1]The original version with Eqs. (1) and (2) was originally submitted to ASCE on September 13, 2001, and an expanded version with Eq. (3) was submitted to ASCE on September 22, 2001. The appendices were added between September 28 and October 5, 2001; The preliminary report (Bažant and Zhou 2001) on which this article is based was posted on September 14, 2001, at ⟨http://www.civil.northwestern.edu/news⟩, ⟨http://www3.tam.uiuc.edu/news/200109wtc⟩, and ⟨http://math.mit.edu/~bazant⟩.

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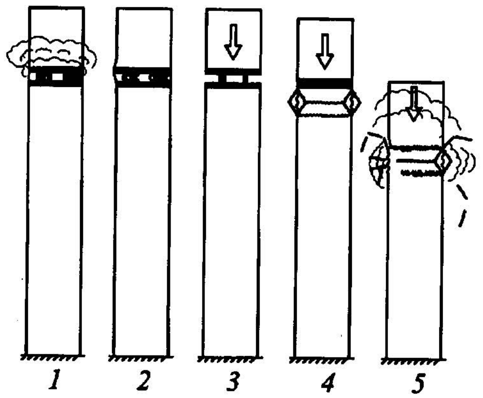

Fig. 1. Stages of collapse of the building (floor height exaggerated)

For our purpose, we may assume that all the impact forces go into the columns and are distributed among them equally. Unlikely though such a distribution may be, it is nevertheless the most optimistic hypothesis to make because the resistance of the building to the impact is, for such a distribution, the highest. If the building is found to fail under a uniform distribution of the impact forces, it would fail under any other distribution. According to this hypothesis, one may estimate that $C \approx 71$ GN/m (due to unavailability of precise data, an approximate design of column cross sections had to be carried out for this purpose).

The downward displacement from the initial equilibrium position to the point of maximum deflection of the lower part (considered to behave elastically) is $h+(P/C)$ where $P=$ maximum force applied by the upper part on the lower part and $h=$ height of critical floor columns (=height of the initial fall of the upper part) ≈ 3.7 m. The energy dissipation, particularly that due to the inelastic deformation of columns during the initial drop of the upper part, may be neglected, i.e., the upper part may be assumed to move through distance h almost in a free fall (indeed, the energy dissipated in the columns during the fall is at most equal to $2\pi \times$ the yield moment of columns, $\times$ the number of columns, which is found to be only about 12% of the gravitational potential energy release if the columns were cold, and much less than that at 800°C). So the loss of the gravitational potential energy of the upper part may be approximately equated to the strain energy of the lower part at maximum elastic deflection. This gives the equation $mg[h+(P/C)]=P^2/2C$ in which $m=$ mass of the upper part (of North Tower) $\approx 58\times 10^6$ kg, and $g=$ gravity acceleration. The solution $P=P_{\mathrm{dyn}}$ yields the following elastically calculated overload ratio due to impact of the upper part:

$$P_{\mathrm{dyn}}/P_0 = 1+\sqrt{1+(2Ch/mg)} \approx 31 \quad (1)$$

where $P_0=mg=$ design load capacity. In spite of the approximate nature of this analysis, it is obvious that the elastically calculated forces in columns caused by the vertical impact of the upper part must have exceeded the load capacity of the lower part by at least an order of magnitude.

Another estimate, which gives the initial overload ratio that exists only for a small fraction of a second at the moment of impact, is

$$P_{\mathrm{dyn}}/P_0 = (A/P_0)\sqrt{2\rho g E_{ef} h} \approx 64.5 \quad (2)$$

where $A=$ cross section area of building; $E_{ef}=$ cross section stiffness of all columns divided by A; and $\rho=$ specific mass of building per unit volume. This estimate is calculated from the elastic wave equation which yields the intensity of the step front of the downward pressure wave caused by the impact if the velocity of the upper part at the moment of impact on the critical floor is considered as the boundary condition (Bažant and Cedolin 1991, Sec. 13.1). After the wave propagates to the ground, the former estimate is appropriate.

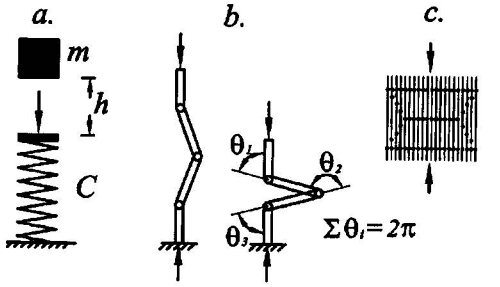

Fig. 2. (a) Model for impact of upper part on lower part of building; (b) Plastic buckling mechanism on one column line; (c) Combination of plastic hinges creating a buckle in the tube wall

Analysis of Inelastic Energy Dissipation

The inelastic deformation of the steel of the towers involves plasticity and fracture. Since we are not attempting to model the details of the real failure mechanism but seek only to prove that the towers must have collapsed and do so in the way seen ("Massive" 2001; *American* 2001), we will here neglect fracture, even though the development of fractures, especially in column connections, is clearly discerned in the photographs of the collapse. Assuming the steel is to behave plastically, with unlimited ductility, we are making the most optimistic assumption with regard to the survival capacity of the towers (in reality, the plastic hinges, especially the hinges at column connections, must have fractured, and done so at relatively small rotation, causing the load capacity to drop drastically).

The basic question to answer is: Can the fall of the upper part be arrested by energy dissipation during plastic buckling, which follows the initial elastic deformation? Many plastic failure mechanisms could be considered, for example: (1) the columns of the underlying floor buckle locally (Fig. 1, stage 2); (2) the floor-supporting trusses are sheared off at the connections to the framed tube and to the core columns and fall down within the tube, depriving the core columns and the framed tube of lateral support, and thus promoting buckling of the core columns and of the framed tube under vertical compression [Fig. 1, stage 4, and Fig. 2(c)]; or (3) the upper part is partly wedged within the emptied framed tube of the lower part, pushing the walls of the framed tube apart (Fig. 1, stage 5). Although each of these mechanisms can be shown to lead to total collapse, a combination of the last two seems more realistic [the reason: multistory pieces of the framed tube, with nearly straight boundaries apparently corresponding to plastic hinge lines causing buckles on the framed tube wall, were photographed falling down, "Massive 2001"; *American* 2001].

Regardless of the precise failure mode, experience with buckling indicates that while many elastic buckles simultaneously coexist in an axially compressed tube, the plastic deformation localizes (because of plastic bifurcation) into a single buckle at a time

[Fig. 1, stage 4, and Fig. 2(c)], and so the buckles must fold one after another. Thus, at least one plastic hinge, and no more than four plastic hinges, per column line are needed to operate simultaneously in order to allow the upper part to continue moving down [Fig. 2(b)] (Bažant and Cedolin 1991). (This is also true if the columns of only one floor are buckling at a time.) At the end, the sum of the rotation angles θ_i ($i=1,2,\ldots$) of the hinges on one column line, $\Sigma\theta_i$, cannot exceed 2π [Fig. 2(b)]. This upper-bound value, which is independent of the number of floors spanned by the buckle, is used in the present calculations since, in regard to survival, it represents the most optimistic hypothesis, maximizing the plastic energy dissipation.

Calculating the dissipation per column line of the framed tube as the plastic bending moment M_p of one column (Jirásek and Bažant 2002) times the combined rotation angle $\Sigma\theta_i=2\pi$ [Fig. 2(b)] and multiplying this by the number of columns, one concludes that the plastically dissipated energy W_p is, optimistically, of the order of 0.5 GN m (for lack of information, certain details such as the wall thickness of steel columns, were estimated by carrying out approximate design calculations for this building).

To attain the combined rotation angle $\Sigma\theta_i=2\pi$ of the plastic hinges on each column line, the upper part of the building must move down by the additional distance of at least one floor below the floor where the collapse started, and so the total release of gravitational potential energy is $W_g=mg\cdot 2h\approx 2\times 2.1$ GN m $=4.2$ GN m. To arrest the fall, the kinetic energy of the upper part, which is equal to the potential energy release, would have to be absorbed by the plastic hinge rotations, i.e., W_p would have to be larger than W_g. Rather,

$$W_g/W_p\approx 8.4 \tag{3}$$

So, even under the most optimistic assumptions by far, the plastic deformation can dissipate only a small part of the kinetic energy acquired by the upper part of building.

When the next buckle with its group of plastic hinges forms, the upper part has already traveled many floors down and has acquired a much higher kinetic energy; the percentage of the kinetic energy dissipated plastically is then of the order of 1%. The percentage continues to decrease further as the upper part moves down. If fracturing in the plastic hinges were considered, a still smaller (in fact much smaller) energy dissipation would be obtained. So the collapse of the tower must be an almost free fall. This conclusion is supported by the observation that the duration of the collapse of each tower, reported as roughly 10 s, was about the same as the duration of a free fall in a vacuum from the tower top $H=416$ m to the top of the heap of debris ($H_0=25$ m), which is $t=\sqrt{2(H-H_0)/g}=8.93$ s. It further follows that the brunt of vertical impact must have gone directly into the columns of the framed tube and the core, and that the front of collapse of the floors could not have advanced substantially ahead of the front of collapse of the framed tube, since otherwise the collapse of the framed tube would have had to take significantly longer than 9 s.

Closing Comments and Problems of Disaster Mitigation

Designing tall buildings to withstand this sort of attack seems next to impossible. It would require a much thicker insulation of steel with blast-resistant protective cover. Replacing the rectangular framed tube by a hardened circular monolithic tube with tiny windows might help to deflect much of the debris of impacting aircraft and the fuel sideways, but regardless of cost, who would want to work in such a building?

The problems appear to be equally severe for concrete columns because concrete heated to such temperatures undergoes explosive thermal spalling, thermal fracture, and disintegration due to dehydration (Bažant and Kaplan 1996). These questions arise not only for buildings supported on many columns but also for the recent designs of tall buildings with a massive monolithic concrete core functioning as a tubular mast. These recent designs use high-strength concrete which, however, is even more susceptible to explosive thermal spalling and thermal fracture than normal concrete. The use of refractory concretes as the structural material invites many open questions (Bažant and Kaplan 1996). Special alloys or various refractory ceramic composites may, of course, function at such temperatures, but the cost would increase astronomically.

It will nevertheless be appropriate to initiate research on materials and designs that would postpone the collapse of the building so as to extend the time available for evacuation, provide a hardened and better insulated stairwell, or even prevent collapse in the case of a less severe attack such as an off-center impact, or the impact of an aircraft containing less fuel.

An important puzzle at the moment is why the adjacent 46-story building, into which no significant amount of aircraft fuel could have been injected, collapsed as well. Despite the lack of data at present, the likely explanation seems to be that high temperatures (though possibly well below 800 °C) persisted on at least one floor of that building for a much longer time than specified by the current fire code provisions.

Appendix I. Elastic Dynamic Response to Aircraft Impact

A simple estimate based on the preservation of the combined momentum of the impacting Boeing 767-200 ($\sim$179,000 kg $\times$550 km/h) and the momentum of the equivalent mass M_{eq} of the interacting upper half of the tower ($\sim 141\times 10^6$ kg$\times v_0$) indicates that the initial average velocity v_0 imparted to the upper part of the tower was only about 0.7 km/h $=0.19$ m/s. The response may be assumed to be dominated by the first free vibration mode, of period T_1. Then the maximum deflection $w_0=v_0T_1/2\pi$. Approximately, $T_1=14$ s, based on estimating (very roughly) the bending stiffness of the tower and approximating it as a vibrating cantilever of a uniform mass distribution. This gives $w_0=0.4$ m, which is well within the range of the elastic behavior of the tower. So it is not surprising that the aircraft impact per se damaged the tower only locally.

The World Trade Center was designed for an impact of a Boeing 707-320 rather than a Boeing 767-200. But note that the maximum takeoff weight of that older aircraft is only 15% less than that of a Boeing 767-200. Besides, the maximum fuel tank capacity of that aircraft is only 4% less. These differences are well within the safety margins of design. So the observed response of the towers proves the correctness of the dynamic design. What was not considered in design was the temperature that can develop in the ensuing fire. Here the experience from 1945 might have been deceptive. That year, a two-engine bomber (B-25), flying at about 400 km/h, hit in fog the Empire State Building (381-m tall, built in 1930) at the 79th floor (278 m above ground)—the steel structure suffered no significant damage, and the fire was confined essentially to one floor (Levy and Salvadori 1992).

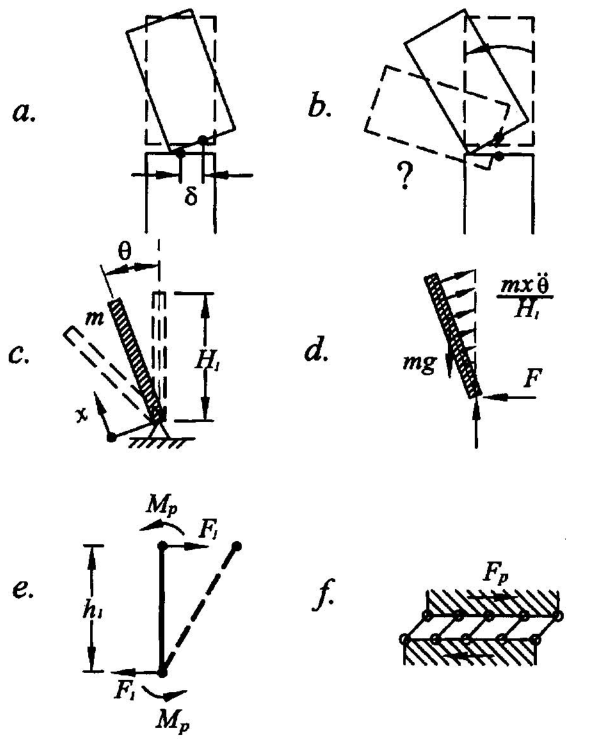

Fig. 3. Pivoting of upper part of tower about its base, (a,b) with and without horizontal shear at base; (c) Model for simplified analysis; (d) Free-body diagram with inertia forces; (d,e) Plastic horizontal shearing of columns in critical floor at base

Appendix II. Why Didn't the Upper Part Pivot About Its Base?

Since the top part of the South Tower tilted [Fig. 3(a)], many people wonder: Why didn't the upper part of the tower fall to the side like a tree, pivoting about the center of the critical floor? [Fig. 3(b)]. To demonstrate why, and thus to justify our previous neglect of tilting, is an elementary exercise in dynamics.

Assume the center of the floor at the base of the upper part [Fig. 3(b)] to move for a while neither laterally nor vertically, i.e., act as a fixed pivot. Equating the kinetic energy of the upper part rotating as a rigid body about the pivot at its base [Fig. 3(c)] to the loss of the gravitational potential energy of that part (which is here simpler than using the Lagrange equations of motion), we have $mg(1-\cos\theta)H_1/2=(m/2H_1)\int_0^{H_1}(\dot{\theta}x)^2dx$ where x=vertical coordinate [Fig. 3(c)]. This provides

$$\dot{\theta}=\sqrt{\frac{3g}{H_1}(1-\cos\theta)},\quad \ddot{\theta}=\frac{3g}{2H_1}\sin\theta \tag{4}$$

where θ = rotation angle of the upper part; H_1 = its height; and the superposed dots denote time derivatives [Fig. 3(c)].

Considering the dynamic equilibrium of the upper part as a free body, acted upon by distributed inertia forces and a reaction with horizontal component F at base [Fig. 3(d)], one obtains $F=\int_0^{H_1}(m/H_1)\ddot{\theta}\cos\theta x\,dx=\frac{1}{2}H_1m\ddot{\theta}\cos\theta=\frac{3}{8}mg\sin 2\theta$. Evidently, the maximum horizontal reaction during pivoting occurs for $\theta=45°$, and so

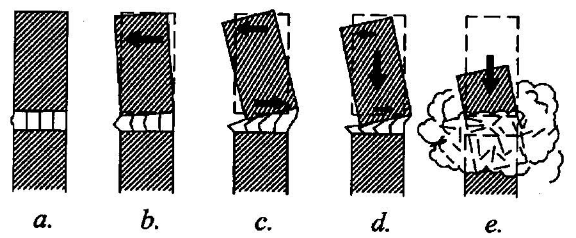

Fig. 4. Scenario of tilting of upper part of building (South Tower)

$$F_{\max}=\tfrac{3}{8}mg=\tfrac{3}{8}P_0\approx 320\,\text{MN} \tag{5}$$

where, for the upper part of South Tower, $m\approx 87\times10^6$ kg.

Could the combined plastic shear resistance F_p of the columns of one floor [Fig. 3(f)] sustain this horizontal reaction? For plastic shear, there would be yield hinges on top and bottom of each resisting column; Fig. 3(e) (again, aiming only at an optimistic upper bound on resistance, we neglect fracture). The moment equilibrium condition for the column as a free body shows that each column can at most sustain the shear force $F_1=2M_p/h_1$ where $h_1\approx 2.5$ m=effective height of column, and $M_p\approx 0.3$ MN m=estimated yield bending moment of one column, if cold. Assuming that the resisting columns are only those at the sides of the framed tube normal to the axis of rotation, which number about 130, we get $F_p\approx 130F_1\approx 31$ MN. So, the maximum horizontal reaction to pivoting would cause the overload ratio

$$F_{\max}/F_p\approx 10.3 \tag{6}$$

if the resisting columns were cold. Since they are hot, the horizontal reaction to pivoting would exceed the shear capacity of the heated floor still much more (and even more if fracture were considered).

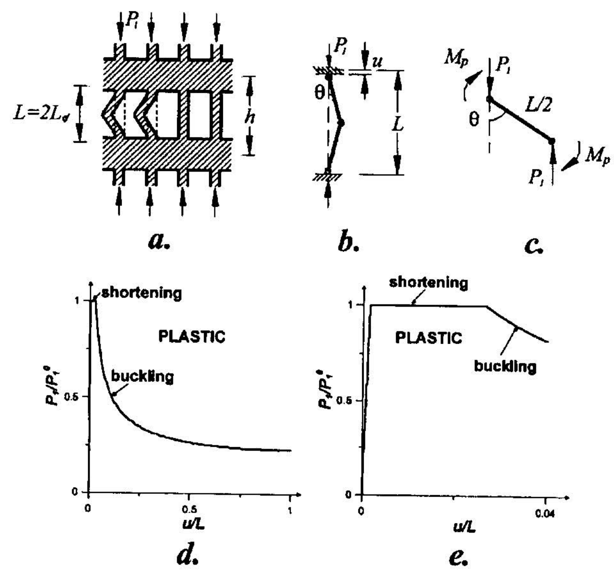

Fig. 5. (a) Plastic buckling of columns; (b) Plastic hinge mechanism; (c) Free-body diagram; (d) Dimensionless diagram of load P_1 versus axial shortening u of columns of the towers if the effects of fracture and heating are ignored; and (e) Beginning of this diagram in an expanded horizontal scale (imperfections neglected)

Since F is proportional to $\sin 2\theta$, its value becomes equal to the plastic limit when $\sin 2\theta = 1/10.3$. From this we further conclude that the reaction at the base of the upper part of South Tower must have begun shearing the columns plastically already at the inclination

$$\theta \approx 2.8° \tag{7}$$

The pivoting of the upper part must have started by an asymmetric failure of the columns on one side of building, but already at this very small angle the dynamic horizontal reaction at the base of the upper part must have reduced the vertical load capacity of the remaining columns of the critical floor (even if those were not heated). That must have started the downward motion of the top part of the South Tower, and afterwards its motion must have become predominantly vertical (Fig. 4). Hence, a vertical impact of the upper part onto the lower part must have been the dominant mechanism.

Finally, note that the horizontal reaction F_{max} is proportional to the weight of the pivoting part. Therefore, if a pivoting about the center of some lower floor were considered, F_{max} would be still larger.

Appendix III. Plastic Load-Shortening Diagram of Columns

Normal design deals only with initial bifurcation and small deflections, in which the diagram of load versus axial shortening of an elasto-plastic column exhibits hardening rather than softening. However, the columns of the towers suffered very large plastic deflections, for which this diagram exhibits pronounced softening. Fig. 5 shows this diagram as estimated for these towers. The diagram begins with plastic yielding at load $P_1^0 = A_1 f_y$ where A_1 = cross section area of one column and f_y = yield limit of steel. At axial shortening 3%, three plastic hinges form as shown in Fig. 5 (if we assume, optimistically, fixed ends). From the condition of moment equilibrium of the half-column as a free body (Fig. 5), the axial load then is $P_1 = 4M_p/L \sin\theta$, while, from the buckling geometry, the axial shortening is $u = L(1-\cos\theta)$, where L = distance between end hinges. Eliminating plastic rotation θ, we find that the plastic load-shortening diagram is given by

$$P_1 = \frac{4M_p}{L\sqrt{1-[1-(u/L)]^2}} \tag{8}$$

which defines the curve plotted in Fig. 5. This curve is an optimistic upper bound since, in reality, the plastic hinges develop fracture (Bažant and Planas 1998), and probably do so already at rather small rotations.

Note Added in Proof

An addendum to this paper will be published in the March 2002 issue of the *Journal of Engineering Mechanics*. An edited manuscript containing the information in the addendum was received by ASCE on October 13, 2001.

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DISCUSSIONS AND CLOSURES

Discussion of "Why Did the World Trade Center Collapse?—Simple Analysis" by Zdeněk P. Bažant and Yong Zhou

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The writers deserve commendation for their lucid analysis of the World Trade Center collapse. The discusser would like to raise the following points.

1. The writers have mentioned that the fracture of hinges at the column connections must have caused the load capacity to drop drastically. More often in many tall buildings the floor-to-floor heights are not uniform and differ considerably at the bottom floors and at certain few top floor levels. Because of the creep buckling and differential shortening of the columns the connections give away, as they cannot resist any secondary moments. Hence, the hinge connections do not influence the failure of the columns and the converse is not true. As such, there is no scope for the formation of a plastic mechanism.

2. The discusser does not agree with the contention of the writers that the walls of the framed tube are pushed apart during the collapse. The structure being a tubular one, there is a complete void near to the center of gravity of the tower. During collapse the core columns will tend to fall inwardly and the perimeter columns also follow suit. Also, the upper part can partly wedge within the emptied framed tube of the lower part, only when the upper floor can fall as a single block. The core columns, floor trusses, and the perimeter columns separate out at collapse and fall mostly as individual units onto the lower floor and wedging is not possible.

3. The aircraft had hit the North Tower between the 90 and 96 floors and the impact was an almost centered one. But, the South Tower was hit by an aircraft between the 75 and 84 floors and the impact was an off-centered one affecting the corner portion of the building heavily. The 78th floor of the South Tower had a sky lobby and could have had a different structural arrangement, with a load capacity lower than that of the other floors. The off-centered impact could have produced a torque, which might have influenced the tilting of the upper part of the South Tower. It is interesting to note that the destroyed floors of the North Tower by direct impact of the aircraft had no sky lobby floor.

4. The columns in the floor that were directly hit by the aircraft lost their capacity to transmit and bear loads any further. Instead, they hung onto the top floors and because of their enormous self-weight exerted a pulling force on the floors above leading ultimately to a pancake failure of the tower. This is evident in the early failure of the South Tower where the number of floors, above the direct hit destroyed floors, is higher than the North Tower.

5. The bending rigidity index (BRI) (Taranath 1998) of the towers is 33, implying a greater flexibility. After the impact of the aircraft, the South Tower because of its flexibility swayed for a duration of around 7–10 s. If the upper part of the South Tower has to pivot about its base, it should have happened during the period of sway, by shifting the center of gravity away by several feet, which is impossible.

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Closure to "Why Did the World Trade Center Collapse?—Simple Analysis" by Zdeněk P. Bažant and Yong Zhou

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Srivakumar's comments are thought provoking and deeply appreciated. However, although his five points introduce interesting connotations, the writers cannot agree with his reservations and objections, for the following reasons.

1. It is not true that "there is no scope for the formation of a plastic failure mechanism." Even though the connection is probably weaker than the column itself, its moment capacity is not zero, which means that the failing connection will not act as a hinge but as a plastic hinge (or fracturing hinge). Moreover, even if the connections are weaker than the columns, the plastic hinges will not necessarily form at the connections because the connections might not in general lie at the locations that create the failure mechanism with the lowest energy dissipation requirement.

2. It was not stated in the paper that "the walls of the framed tube are pushed apart during collapse." What was stated in the paper is that one may consider the *possibility* that "the upper part is partly wedged within the emptied framed tube of the lower part, pushing the walls of the framed tube apart" (p. 2). The writers cannot agree with the statement that the "wedging is not possible." This possibility cannot be excluded. But the point is anyway extraneous. The wedging was not considered in the analysis because the stated aim was to prove that the towers must have collapsed. For that purpose, the most optimistic assumptions about the structure resistance had to be made, and the assumption of wedging would not be of that kind.

3. While it is of course true that the off-center impact of aircraft into the South Tower must have "produced a torque," this torque could have affected only the initial vibrations of the tower lasting less than a minute and could not have had any effect on

the collapse which occurred much later. The tilting of the upper part observed during the collapse must have been caused by anoff-center hole in the building but not by the initially produced torque.

4. It is dubious to say that "the columns in the floor that were directly hit by the aircraft." ... "hang on top of the floors and because of their enormous self-weight exert a pulling force on the floors above." They of course exert some force, but compared to the load from the upper part this force is negligible.

5. The writers agree that the pivoting of the upper part of the South Tower about its base was not possible during the initial period of swaying after the impact. But, what the analysis addressed was the question of pivoting during the collapse, which was shown to have little effect.